Young Researchers Workshop on
Dynamical Systems and Optimal Control

Madeira, 21-23 January 2012

Mini courses:

- María Soledad Aronna, Università degli Studi di Padova
  "The Pontryagin Maximum Principle and its proof"
- Cédric M. Campos, Technische Universität Munchen
  "Geometric Dynamics and Discretization"
- Adriano Festa, Imperial College London
  "Introduction to Differential Games"
- Francisco Silva, Université de Limoges
  "Mean Field Games"
- Daniela Tonon, Université Pierre et Marie Curie
  "Hamilton-Jacobi equations, viscosity solutions and semiconcave functions"

Contributed talks:

- Andrea Boccia, Imperial College London
  "The Pontryagin Maximum Principle and State Constraints"
- Daniel Höhener, Université Pierre et Marie Curie
  "Tools of Set-Valued Analysis and Perturbations of Trajectories"
- Juan Pablo Maldonado, Université Pierre et Marie Curie
  "Repeated games in discrete time"
- Michele Palladino, Imperial College London
  "Relaxation for Optimal Control Problems"
- Mario Zanon, Katholieke Universiteit Leuven
  "Review on direct methods for optimal control"

Programme:

	Monday
10:00 - 10:30	Opening session
10:30 - 12:30	F. Silva
12:30 - 14:00	Lunch
14:00 - 14:30	P. Maldonado
14:30 - 15:00	Coffee break
15:30 - 17:30	A. Festa

Tuesday

09:00 - 11:00 C. Campos
11:00 - 11:30 Coffee break
11:30 - 12:30 M. Zanon
12:30 - 14:00 Lunch
14:00 - 16:00 D. Tonon

Wednesday
09:00 - 11:00 S. Aronna
11:00 - 11:30 Coffee break
11:30 - 13:30 A. Boccia, D. Höhener, M. Palladino

Abstracts

María Soledad Aronna, Università degli Studi di Padova
"The Pontryagin Maximum Principle and its proof"
In this course we present various formulations of the Pontryagin Maximum Principle. In particular, we consider problems both without and with endpoint constraints. First we review the proof of the simplest formulation of the principle to get the idea behind it, without the technicalities and complications of the general case. Afterwards, we study the proof of the general case.

Andrea Boccia, Imperial College London
"The Pontryagin Maximum Principle and State Constraints"
Our goal is to extend the earlier derived necessary conditions of optimality, in the form of a Maximum Principle, to allow for pathwise constraints on the state trajectories. There are several different strategies for deriving necessary conditions of optimality when state constraints are present. Most of them are direct and do not hinge on the simpler Maximum Principle in absence of such constraints. We here want to explore the possibility of reduce the original problem into the state constraint-free case and then use known necessary conditions of optimality.

Cédric M. Campos, Technische Universität Munchen
"Geometric Dynamics and Discretization"
We will review the different geometric formulations of Classical Mechanics and Field Theory and derive the variational principles within. From there, we will recover the different dynamical equations, Euler-Lagrange equations, Hamilton's equations, Hamilton-Jacobi equation and uncover the key ideas of a geometric discretization.

Adriano Festa, Imperial College London
"Introduction to Differential Games"
The term "Differential Games" is applied to a group of problems in applied mathematics that share certain characteristics related to the modeling of conflict. In a basic differential game there are two actors -- a pursuer and an evader -- with conflicting goals. The pursuer wishes, in some

sense, to catch the evader, while the evader's mission is to prevent this capture. These "games" are modeled mathematically by first defining state variables that represent the position (and perhaps velocity) of the participants, determining (differential) equations of motion for the rivals, and then describing sets in the state space called target sets. (For example, a target set for a pursuer may include points in the state space where the distance between the pursuer and the evader is small.) Each participant in the game tries to drive the state variables of the game into a particular target set by controlling key variables called, naturally, controls. The study of these games has several implications in many field of economics, artificial intelligence, robot navigations. We will focus on the Isaacs-Bellman approach (involving a first order non linear PDE) and in the following round table we will discuss together some aspects and some applicative situations where this theory can be applied.

Daniel Höhener, Université Pierre et Marie Curie
"Tools of Set-Valued Analysis and Perturbations of Trajectories"
Set-valued analysis is an essential tool in control theory. We start by presenting some of the most popular notions such as tangent- and normal cones and show the links between these notions. The second part is concerned with differential inclusions. The main focus lies on a description of the solution set of a perturbed differential inclusion. We end with some examples of applications of this result.

Juan Pablo Maldonado, Université Pierre et Marie Curie
"Repeated games in discrete time"
In this talk we will present a brief introduction to the theory of repeated games in discrete time. We will present several examples to illustrate the phenomena we are interested in (mostly on the two player, zero sum setting) and we will discuss briefly some links between repeated and differential games.

Michele Palladino, Imperial College London
"Relaxation for Optimal Control Problems"
Relaxation is a procedure in optimal control problem in which extra elements are added to the domain of an optimization problem in order to guarantee the existence of a minimizer. Of course the relaxed problem, to be of interest, must retain a close relation with the original problem. In this talk, we will give a general survey about the link between relaxation procedure and theoretical/applied applications. Furthermore we will discuss the case when some pathologies in the relaxation procedure arise and the relation between this phenomenon and the Pontryagin Maximum Principle.

Francisco Silva, Université de Limoges
"Mean Field Games"
TBA

Daniela Tonon, Université Pierre et Marie Curie

"Hamilton-Jacobi equations, viscosity solutions and semiconcave functions"

We present the general theory of Hamilton-Jacobi equations of the type $u_t+ H(t,x,D_x u)=0$ in an open set of $\mathbb{R}^(n+1)$. Using the method of characteristic we show why viscosity solutions need to be introduced and what are their properties. Particular attention is devoted to the fact that the viscosity solution is unique and to its representation formula. We introduce semiconcave functions, we describe their properties and show how they are related to Hamilton-Jacobi equations. We show that semiconcave initial data preserve this property in the evolution and that in the case of a strictly convex Hamiltonian the viscosity solution becomes semiconcave due to a regularizing effect.

Mario Zanon, Katholieke Universiteit Leuven

“Review on direct methods for optimal control”

Direct methods for optimal control are also called "first discretize, then optimize". The system dynamics are first discretized by integrating in time the ODE or DAE modeling the plant. The optimal control problem is thus translated into a nonlinear programming problem that can be solved with efficient algorithms. In the talk the steps that lead from the formulation of the problem to its solution will be presented.