SADCO Internal Meeting

Madeira, Portugal, January 24- 25, 2013

Speaker: Maria Soledad Aronna (University of Padova)
Title: Necessary conditions involving Lie brackets for impulsive optimal control problems
(joint work with Franco Rampazzo)
Abstract
We study control problems for systems that are governed by ordinary differential equations whose vector fields depend linearly on the time derivatives of some components of the control; which we refer as "impulsive controls." We assume that the vector fields multiplying the derivatives of the impulsive controls are commutative. We use a concept of solution for these impulsive differential equations that was introduced by A. Bressan and F. Rampazzo in the 90's. They showed that an impulsive system can be reduced to a classical system of ordinary differential equations via a transformation of variables. We extend their results for our case with both impulsive and classical controls. For an optimal control problem in the Mayer form and with final state constraints, we prove a maximum principle and higher-order necessary conditions in terms of the adjoint state and some Lie brackets of the involved vector fields.

Speaker: Frédéric Bonnans (Inria Saclay)
Title: Optimal control of a semilinear parabolic equation with singular arcs

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Speaker: Annalisa Cesaroni (University of Padova)
Title: Multiscale financial models with stochastic volatility and singular perturbations problems
Abstract
I will discuss some singular perturbations problems for partial integro-differential equations arising from models of pricing and trading derivative securities in financial markets with stochastic volatility.

Speaker: Francis Clarke (Université de Lyon)
Title: Error and fallacy in control
Abstract
The talk examines the various types of mistakes that occur in both optimal and stabilizing control.

Speaker: Italo Capuzzo Dolcetta (Sapienza Universitá di Roma)
Title: Some topics in mean field games
Abstract
I will review some known and more recent results concerning partial differential problems arising in the Lasry-Lions approach to mean field games. I will focus in particular on the deterministic basic model

$$\inf \int_t^T \left[\frac{1}{2}|\dot{X}_s|^2 + L(X_s, m(s, X_s))\right] ds + G(X_T, m(T, X_T)) \,.$$

Here, m is a non-negative density function valued in $[0,1]$ whose time evolution of m starting from an initial configuration $m(0,x)$ is governed by the continuity equation

$$\partial_t m(t,x) - \operatorname{div}\Big(m(t,x) D_x u(t,x)\Big) = 0 \quad \text{in } (0,T)\times \mathbb{R}^d\,.$$

Speaker: Lars Grüne (Universität Bayreuth), Tobias Damm (Universität Kaiserslautern), Marleen Stieler and Karl Worthmann (both Universität Bayreuth)
Title: Sufficient conditions for the exponential turnpike property
Abstract
The turnpike property is a classical property of finite horizon optimal trajectories which describes the fact that the optimal trajectories stay in a neighborhood N of an equilibrium for a while, where N shrinks down to the equilibrium as the optimization horizon tends to infinity. In this talk the exponential turnpike property is considered, in which the diameter of N shrinks exponentially fast as the optimization horizon tends to infinity. The motivation for this stronger version of the turnpike property stems from the fact that on the one hand it can be observed numerically for a number of examples and on the other hand it plays an important role in the analysis of Model Predictive Control schemes, which will be briefly explained. We will then show that a dissipativity property (which is well known in turnpike theory) together with suitable bounds on appropriately defined optimal value functions imply the exponential turnpike property. Moreover, we show how stabilizability and controllability properties can be used in order to infer these bounds.

Speaker: Frédéric Jean (ENSTA ParisTech)
Title: Goal-oriented locomotion: the inverse optimal control approach

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Speaker: Urszula Ledzewicz, (Southern Illinois University Edwardsville, USA)
Title: Challenges in Synthesis of Optimal Controls For Problems Arising in Biomedicine
Abstract
In the talk, I will present applications of geometric methods in optimal control theory to the analysis of models for various cancer treatments. In some cases, like a class of models for anti-angiogenic treatment, a complete optimal synthesis of controlled trajectories can be constructed. A geometric analysis of trajectories near an optimal singular arc (including saturation effects) reveals the structure of optimal controls as specific concatenations of bang-bang controls (representing therapies of full dose with rest periods) and singular controls (therapies with specific time-varying partial doses). However, even for this model things become more difficult when additional treatments like chemotherapy or radiotherapy are incorporated into the model leading to multi-input optimal control problems. Singular controls also arise as a natural choice for the optimal controls in models representing other treatments like the combination of chemo- and immunotherapy. Using tools of geometric control theory that are based on Lie bracket computations and high-order necessary conditions for optimality, generally analytic formulas for optimal singular controls and associated singular arcs can be computed, but determining their concatenation structures with bang controls becomes a challenge in high dimensions. Even for the single input problem representing a mono-treatment therapy these concatenations become complicated when the model is extended to include the pharmacokinetics of the drugs as an additional dynamics. This fact raises the order of the singular arc and generates chattering phenomena.
In spite of the simplicity of the models, the results of this analysis in many aspects agree with medical experiments. Thus these mathematical challenges are also of medical importance in view of the growing awareness that the dosages and timing of the administration of the anti-cancer drugs can have an impact on the outcome of the treatment. This has led to the development of a worldwide Metronomics Health Initiative

whose aim is instead of searching for new anti-cancer drugs, to search for better ways of administering the existing ones.

Speaker: Fernando Fontes (Universidade do Porto, FEUP)
Title: Integral-type Constraint Qualification for Degenerate Maximum Principle
Abstract
A new constraint qualification under which a nondegenerate maximum principle is validated is proposed. In contrast with existing results, this constraint qualification is of an integral type.

Speaker: Daniel Hoehener (Université Pierre et Marie Curie)
Title: Second-Order Necessary and Sufficient Optimality Conditions for Constrained Control Problems.

Speaker: Paola Loreti (Sapienza Universitá di Roma)
Title: Some aspects of Hopf-Lax-Oleinik type formulas
Abstract
Inspired by the theory of idempotent analysis, we discuss some basic notions in an idempotent setting, then, in this framework, we discuss some results on a class of Hamilton-Jacobi equations. The talk is based on joint works with A. Avantaggiati.

Speaker: Helmut Maurer (Universität Münster) Tanya Tarnopolskaya (CSIRO Mathematics. Informatics and Statistics, Sydney), Neale Fulton (CSIRO Mathematics. Informatics and Statistics, Canberra)
Title: Computation of bang-bang and singular controls in planar collision avoidance
Abstract
We study optimal cooperative collision avoidance strategies for two participants in a planar close proximity encounter. Previous research focused on special cases of this problem and showed that bang-bang strategies without switching are optimal in most situations, while singular controls only appear for the case of participants with unequal linear speeds under certain conditions. In this talk, we consider the general case of coplanar close proximity encounter and present a theoretical and numerical study of the structure of optimal controls. It is shown that both controls can not be singular simultaneously and that the only possible singular control is a zero control for the slower participant. We derive formulas for the singular surfaces and verify that sufficient conditions hold for the computed extremal solutions. Different types of structural changes of the control strategies are identified and the transition point between strategies are computed.

Speaker: Sabine Pickenhain (Brandenburg Technical Universtity)
Title: Optimal Control Problems with Infinite Horizon - a Hilbert Space Approach
Abstract
We consider a class of infinite horizon optimal control problems as optimization problems in Hilbert spaces. This special class of problems arises in the theory of economic growth and in processes where the time T is an exponentially distributed random variable. Known results about the validity of Pontryagin's Maximum Principle use an approximation approach. We propose a complete different one. We show that in typical applications the state and control variable belongs to the Weighted Sobolev space $W_2^1(\mathbb{R}^+, \nu)$ and Lebesgue space $L_2(\mathbb{R}^+, \nu)$ respectively. In this spaces the problem can be treated by Hilbert space methods. Making appropriate assumptions on the growth of the data of the problem we can prove Pontryagin's Maximum Principle as a separation theorem in Hilbert spaces. In contradiction to the first approach the obtained

maximum principle includes also transversality conditions and the existence of optimal solutions can be guaranteed. For problems involving dynamics linear with respect to the state we demonstrate that necessary optimality conditions and adjoints depend on transformations. A logarithmic transformation is used to transform the state and to obtain a new equivalent class of problems where the right hand side of differential equation does not depend on the state any more. Besides of this, the state function fulfills so called natural state constraints pointwisely which assures the boundedness of the feasible set in the norm of the considered spaces and plays an important role for guaranteeing the existence of an optimal solution. For the transformed problem it is possible to apply a Pontryagin type Maximum Principle and an existence theorem. Some logarithmic transformation of the time domain is applied to transform the considered infinite horizon control problem into a one with finite horizon. Through equivalence of these two problems one is able to obtain a Pontryagin type Maximum Principle for control problems with finite horizon but with discontinuities both in the integrand and the differential equation.

Speaker: Sylvain Sorin (Université Pierre et Marie Curie)
Title: On Some Recent Links Between Discrete and Continuous Time Repeated Games

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Speaker: Daniela Tonon (Université Pierre et Marie Curie)
Title: Pointwise second order necessary conditions for optimality in optimal control problems with constraints on controls

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Speaker: Fabian Wirth (Universität Würzburg)
Title: Stability of SIS Epidemiological Models under Switching
Abstract
We study the spread of disease in an SIS model on a graph. The model considered is a time-varying, switched model, in which the parameters of the SIS model are subject to abrupt change. It is shown that the joint spectral radius can be used as a threshold parameter in the spirit of the basic reproduction number for time-invariant models.

Speaker: Hasnaa Zidani (ENSTA ParisTech & Inria Saclay)
Title: Hamilton-Jacobi theory for optimal control problems with state constraints