Control of coupled fast and slow dynamics

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Abstract

Through singularly perturbed differential equations, we examine the behavior of a fast motion coupled with a slow one, when both are subject to control. Of interest are issues of stabilizing design and optimal control, including the relevant sensitivity analysis. We focus on the case where the fast dynamics does not necessarily exhibit a stationary limit behavior. Rather, invariant measures that give rise to Young measures portray the desired limit. The case were the slow and fast motions are not depicted by separated variables will be discussed. Relations to the methods of homogenization and functional convergence will be displayed. Motivation, stemmed from natural and from industrial needs, will be alluded to, as well as computational aspects.

Essential references
The references point to the tools used, rather than to the material itself.

1. C. Castaing, P. Raynaud de Fitte and M. Valadier, Young measures on topological spaces. With applications in control theory and probability theory. Mathematics and its Applications, 571. Kluwer Academic Publishers, Dordrecht, 2004.

2. P. Pedregal, Parameterized Measures and Variational Principles. Birkhäuser Verlag, Basel, 1997

3. M. Valadier, *A course on Young measures*, Rend. Istit. Mat. Univ. Trieste 26 (1994) supp., 349-394.

Optimal control, Hamilton-Jacobi equations and singularities in euclidean and riemaniann spaces

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Abstract

Lecture 1. Calculus of variations and optimal control problems. Differential inclusions. Existence of solutions to optimal control problems. Necessary optimality conditions: Euler-Lagrange equations and Pontryagin maximum principle.

Lecture 2. Value function and dynamic programming principle. Lipschitz continuity of the value function. Hamilton-Jacobi equation. Generalized differentials. Viscosity solutions. Uniqueness and comparison results.

Lecture 3. Semiconcavity of the value function. Semiconcave functions, reachable gradients and superdifferentials. Singular sets of semiconcave functions. Rectifiability. Propagation of singularities. Propagation along generalized characteristics. Invariance of the singular set under the generalized gradient flow. Homotopy equivalence.

Lecture 4. Riemanniann manifolds, parallel transport, geodesics. Semiconcave functions on riemannian manifolds and their generalized gradients. Propagation of singularities along the generalized gradient flow of the riemannian distance. Homotopy equivalence and time optimal control problems.

Essential references

1. M. Bardi M., I. Capuzzo Dolcetta , Optimal control and viscosity solutions of Hamilton–Jacobi equations, Birkhäuser, Boston, 1997.

2. P. Cannarsa P., C. Sinestrari, Semiconcave functions, Hamilton-Jacobi equations, and optimal control, Birkhäuser, Boston, 2004.

3. P.L. Lions , Generalized solutions of Hamilton-Jacobi equations, Pitman, Boston, 1982.

4. J. Zabczyk J., Mathematical control theory: an introduction. Reprint of the 2nd corrected printing 1995. (English) Modern Birkhäuser Classics. Boston, MA: Birkhäuser. xii, 260 p. (2008).

Optimal control and mean field games

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Abstract

Mean field games are used to understand the behavior of a very large number of identical agents, each individually trying to optimize their position in space and time, but with their preferences being partly determined by the choices of all the other agents. The simplest model to describe this phenomenon is the coupling of a Hamilton-Jacobi equation—modeling the behavior of each agent—with a Fokker-Planck equation—which governs the evolution of the agents' density. This model has been introduced by Lasry and Lions in 2005 and widely used since then. In these lectures we will mainly concentrate on deterministic mean field games and explain, on the one hand, how to use semi-concavity properties of optimal control problems to solve the mean field game system and, on another hand, how this system is related with the optimal control of Hamilton-Jacobi equations.

Essential references

1. P. Cardaliaguet, *Notes on mean field games (from P.-L. Lions' lectures at Collège de France)*,
   http://www.ceremade.dauphine.fr/∼cardalia/MFG100629.pdf
2. J.M. Lasry,, P.L. Lions, *Mean field games*, Japan J. Math. 2 (2007), no. 1, 229–260.
3. J.M. Lasry,, P.L. Lions, *Jeux à champ moyen. I. Le cas stationnaire.* C. R. Math. Acad. Sci. Paris 343 (2006), no. 9, 619–625.
4. J.M. Lasry,, P.L. Lions, *Jeux à champ moyen. II. Horizon fini et contrôle optimale*, C. R. Math. Acad. Sci. Paris 343 (2006), no. 10, 679–684.

Quantum control

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Abstract

Recent theoretical and experimental advances mean that it is now possible to control physical systems at the quantum level. Indeed, developments in quantum technology require the control of quantum systems. These lectures will provide an introduction to both open loop and closed loop (feedback) quantum control.

Lecture 1
Quantum technology, postulates of quantum mechanics, quantum probability.

Lecture 2
Open loop quantum control, unitary group, optimal control, viscosity solutions, numerical methods.

Lecture 3
Closed loop quantum control, open systems, feedback, optimal measurement feedback, coherent feedback, quantum networks.

Essential references
Recent texts on quantum control: [3], [6]. Some survey papers: [1], [2], [4], [5], [7].

1. L. Bouten, R. van Handel, and M.R. James. *An introduction to quantum filtering*, SIAM J. Control and Optimization, 46(6):21992241, 2007.
2. L. Bouten, R. van Handel, and M.R. James, *A discrete invitation to quantum filtering and feedback control*, SIAM Review, 51(2):239316, 2009.
3. D. D'Alessandro, *Introduction to Quantum Control and Dynamics*, Chapman and Hall/CRC, 2007.
4. D. Dong and I.R. Petersen, *Quantum control theory and applications: A survey*, arxiv:0910.2350 quant-ph, 2009.
5. M.R. James and R. Kosut, *Quantum estimation and control*, in W.S. Levine, editor, The Control Handbook. CRC Press, 2010.
6. H.M. Wiseman and G.J. Milburn, Quantum Measurement and Control, Cambridge University Press, Cambridge, UK, 2010.
7. G. Zhang and M.R. James, *Quantum feedback networks and control: A brief survey*, arxiv:1201.6020v1 quant-ph 1912.

Intelligent groups and sparse controls

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Abstract

By intelligent group we mean a collection of independent agents, who act dynamically taking decisions which affect the whole system. Possible applications include: vehicular traffic, pedestrians and crowd dynamics, animal groups, networked robots.
The mathematical models proposed for these systems range both in scales (micro, meso and macro) and math tools (optimization, ODEs, PDEs, etc.). For instance various conservation law models were proposed for vehicular traffic, while social force ODE models were presented for pedestrians.
We focus on recent mixed models, involving continuous-discrete spaces and ode-pde systems, and models coupling. Then a multiscale approach using time evolving measures is showed, with applications to crowd dynamics. The control of such groups can be performed at different scales as well. We will focus the attention on two specific cases, the first of PDE and mixed type, the second of ODE type:
- control of pde models and mixed ode-pde for vehicular traffic and supply chains.
- sparse control of large groups for biological and robotic applications.

Essential references

1. M. Garavello, B. Piccoli, *Conservation laws on complex networks*. Ann. Inst. H. Poincaré Anal. Non Linèaire 26 (2009), no. 5, 1925 V1951.
2. M. Garavello, B. Piccoli, Traffic flow on networks. Conservation laws models. AIMS Series on Applied Mathematics, 1. American Institute of Mathematical Sciences (AIMS), Springfield, MO, 2006. xvi+243 pp.
3. C. D'Apice, S. Gottlich, M. Herty, B. Piccoli, Modeling, simulation, and optimization of supply chains. A continuous approach. Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA, 2010. x+206 pp.
4. B. Piccoli, A. Tosin, *Time-evolving measures and macroscopic modeling of pedestrian flow*. Arch. Ration. Mech. Anal. 199 (2011), no. 3, 707 V738
5. E. Cristiani, P. Frasca, B. Piccoli, *Effects of anisotropic interactions on the structure of animal groups*, J. Math. Biol. 62 (2011), no. 4, 569 V588