SADCO Doctoral Days 2013

Density issues for impulsive controls

M. Soledad Aronna (joint work with Franco Rampazzo)
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Abstract

We consider control systems governed by nonlinear O.D.E.s of the form:

$$\dot{x}(t) = f(x(t), u(t), v(t)) + \sum_{\alpha=1}^{m} g_\alpha(x(t), u(t)) \dot{u}_\alpha(t), \quad \text{for } t \in [0, T],$$

where $x : [0,T] \to \mathbb{R}^n$ is the *state variable,* $u : [0,T] \to \mathbb{R}^m$ is the *impulsive control* and $v : [0,T] \to \mathbb{R}^l$ is the *ordinary control.* The control u is allowed to be an L^1-function, which gives the system an impulsive character. A robust notion of solution for this class of differential equations, already proposed in the literature, is here adopted and slightly generalized to the case where an ordinary, bounded control is present in the dynamics as well. For a problem in the Mayer form we then investigate the question whether this notion of solution provides a "proper extension" of the standard problem with absolutely continuous controls u. Furthermore, we show that this impulsive problem is a variational limit of problems corresponding to controls u with bounded variation.

Free time optimal control problems with time delays

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Abstract

Solutions to optimal control problems for retarded systems, on a fixed time interval, satisfy a form of the Maximum Principle, in which the co-state equation is an advanced differential equation. In this paper we present an extension of this well-known necessary condition of optimality, to cover situations in which the data is non-smooth, and the final time is free. The fact that the end-time is a choice variable is accommodated by an extra transversality condition. A traditional approach to deriving this extra condition is to reduce the free end-time problem to a fixed end-time problem by a parameterized change of the time variable. This approach is problematic for time delay problems because it introduces a parameter dependent time-delay that is not readily amenable to analysis; to avoid this difficulty we instead base our analysis on direct perturbation of the end-time. Formulae are derived for the gradient of the minimum cost as a function of the end-time. It is shown how these formulae can be exploited to construct two-stage algorithms for the computation of solutions to optimal retarded control problems with free-time, in which a sequence of fixed time problems are solved by means of Guinn's transformation, and the end-time is adjusted according to a rule based on the earlier derived gradient formulae for the minimum cost function.

A decomposition technique for pursuit evasion games with many pursuers

Adriano Festa
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Abstract

Restrictions on memory storage impose ultimate limitations on the dimensionality of differential games problems for which optimal strategies can be computed via direct solution of the associated Hamilton-Jacobi-Isaacs equations. It is of interest therefore to explore whether, for certain specially structured differential games of interest, it is possible to decompose the original problem into a family of simpler, lower dimensional, differential games. In this talk we exhibit a class of single pursuer-multiple evader games for which a reduction in complexity of this nature is possible. The target set is expressed as a union of smaller, sub-target, sets. The individual differential games in the family are obtained from the original problem by taking as target set an element in the family of sub-target sets, in place of the original target set. We can exploit geometric features of the dynamic constraints and constraints of the problems arising in this way to reformulate them as lower dimensional, simpler to solve, problems. We give conditions under which the value function of the original can be characterized as the lower envelope of the value functions for the simpler problems and how optimal strategies can be constructed from those for the simpler problems. The methodology is illustrated by several examples.

Controllability of parabolic equations: new results and further applications

Roberto Guglielmi
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Abstract

We introduce well-established results about the controllability of uniform parabolic equations through an additive control and recent advances in the multiplicative control case. Moreover, we show new results concerning the controllability of degenerate parabolic equations. Finally, we present a recent application to the control of stochastic processes and expected future developments.

Infinite Horizon Problems on Stratifiable State Constraints Sets

Cristopher Hermosilla
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Abstract

When dealing optimal control problems with state constraints it is usual to assume some pointing qualification hypothesis. These kind of conditions are imposed to obtain a characterization of the value function as the unique solution to a Hamilton-Jacobi equation among a certain class of functions. Nevertheless, this type of assumptions are quite restrictive and do not allow to treat, for instance, cases where the only option for the trajectories is to stay on the boundary of the set of constraints. In this work we attempt to cover this kind of situations by assuming that the state constraints set admits a sufficiently regular partition into smooth manifolds of different dimensions. We center our study in the value function for the infinite horizon problem and we present a characterization of this function as the unique l.s.c. solution to a bilateral Hamilton-Jacobi equation written in terms of the stratification.

An accelerated semi-Lagrangian/policy iteration scheme for the solution of dynamic programming equations

Dante Kalise
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Abstract

We present some recent results concerning the efficient numerical approximation of static Hamilton-Jacobi-Bellman equations of the form

$$\lambda u(x) + \sup_{a \in \mathscr{A}} \{-f(x,a) \cdot Du(x) - g(x,a)\} = 0, \qquad x \in \mathbb{R}^n,$$

characterizing the value function $u(x)$ of an optimal control problem in $\mathbb{R}^n$. One of the main challenges in the solution of this equation relates to its high-dimensionality, and therefore the design of efficient methods turns to be a fundamental task.

In this talk we present a scheme based on a semi-Lagragian/finite differences discretization [2] combined with an iterative scheme in the space of policies [1, 3]. Moreover, we exploit the idea that a reasonable initialization of the policy iteration procedure yields a faster numerical convergence to the optimal solution. For such purpose, the scheme features a pre-processing step with value iterations in a coarse grid. A series of numerical tests, spanning a wide variety of applications, assess the robust and efficient performance of the method.

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Mixed constrained control in the framework of differential inclusions

Igor Kornienko
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Abstract

Properties of control systems described by differential inclusions are well established in the literature. Of special relevance to optimal control problems are properties concerning measurability, convexity, compactness of trajectories and Lipschitz continuity of the setvalued mapping defining the differential inclusion of interest. In this presentation we focus on dynamic control systems coupled with mixed state-control constraints. We describe a class of such systems with the help of an appropriate differential inclusion so as to preserve ”good” properties of the set-valued mapping.

Localy input-to-state stable Lyapunov function constructed by linear programming

Huijuan Li
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Abstract

In this presentation, we talk about a numerical algorithm for computing a local ISS Lyapunov function for systems which are locally input-to-state stable (ISS). The algorithm relies on two linear programming problems and computes a local ISS Lyapunov function on a triangulation of compact sets excluding a small neighborhood of the origin. If systems which are locally ISS have a C^2 ISS Lyapunov function, then there exists a triangulation such that linear programming problems by the algorithm provides a local ISS Lyapunov function which is linearly affine on each simplex.

Singular Perturbations in Deterministic and Stochastic Control

Joao Meireles
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Abstract

I will discuss some singular perturbations of a class of stochastic and deterministic control problems. Under assumptions, the convergence of the value function is characterized and the methods used are of the theory of viscosity solutions and of the homogenezation of fully nonlinear PDEs. I will show some examples and explain the new directions of my research.

Asymptotic Behavior of Hamilton-Jacobi-Bellman Equation and Infinite Horizon Problem

Nguyen Ngoc Quoc Thuong
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Abstract

In this talk, we present the asymptotic problem for Hamilton-Jacobi-Bellman equation satisfied by the value function associated to the singularly perturbed optimal control problem. To study this topic, we first consider the usual infinite horizon problem with compact control set, say A. The differences are that the cost ℓ has superlinear growth at infinity with respect to the state variable, and the discount factor is zero. Moreover, the local bounded-time controllability on the nonlinear control system is assumed. With this setting, we have obtained some new results on the finiteness and continuity of the value function. An important fact is that, the trajectory is contained in some compact set provided that the corresponding cost and value function is finite. We also consider the optimization problem with a family of running costs $\ell + k$, where k is a real parameter. Because this cost, and hence the associated value function, are infinite, our aim is to concentrate on a real number, say c, such that the corresponding cost and value function are finite. With suitable assumptions, following the dynamic programming principle approach, this value function should satisfy a Hamilton-Jacobi-Bellman equation. Within this framework, some other characteristics of our optimal control problem as well as viscosity solution properties of the Hamilton-Jacobi-Bellman equations are also investigated.

On The Differentiability Of Minimum Time Function For Linear Control Systems

Nguyen Van Luong
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Abstract

Consider the linear control system $\dot{x} = Ax + bu$, where $x \in \mathbb{R}^N$, $u \in [-1,1]$, $A \in \mathbb{M}_{N\times N}(\mathbb{R}^N)$, $b \in \mathbb{R}^N$ satisfy the Kalman rank condition

$$\operatorname{rank}[b, Ab, \ldots, A^{N-1}b] = N$$

We study the minimum time function to reach the origin which is defined by $T(x) = \inf\{t \geq 0 : x \in \mathscr{R}(t)\}$, where $\mathscr{R}(t)$ is the reachable set at time $t \geq 0$,

$$\mathscr{R}(t) = \left\{ x = \int_0^t e^{-As} bu(s)dt \,|\, u : [0,\infty) \to [-1,1] \, \text{is measurable} \right\}$$

Fix $T > 0$ and let $x \in \partial\mathscr{R}(T)$. If ζ is a nonzero normal vector to $\mathscr{R}(T)$ at x then the unique optimal control steering x to the origin in time T is $u_x(t) = -\operatorname{sign}(\langle \zeta, e^{-At}b \rangle)$.

It is known that in this case $T(x)$ admits a second order Taylor expansion a.e. x. Therefore, it is possible to differentiate the Hamilton - Jacobi equation

$$\langle Ax, DT(x) \rangle + |\langle b, DT(x) \rangle| = 1 \tag{1}$$

at a.e. x such that $\langle b, DT(x) \rangle \neq 0$. We obtain the system of PDE's

$$ADT(x) + AxD^2T(x) + \operatorname{sign}(\langle b, DT(x) \rangle) bD^2T(x) = 0 \tag{2}$$

Since $DT(x)$ is never 0, it is a normal vector to $\mathscr{R}(T(x))$ at x. Therefore, solving (1) and (2) gives hope to design a robust synthesis.

To this aim, we prove that T is differentiable on an open set, whose complement is countably $\mathscr{H}^{N-1}$ - rectifiable. Moreover, we give condition which ensure that the reachable set is eventually smooth.

Properties of minimizers that are not also relaxed minimizers

Michele Palladino
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Abstract

Relaxation is a procedure in optimal control problem in which extra elements are added to the domain of an optimization problem in order to guarantee the existence of a minimizer. Of course the relaxed problem, to be of interest, must retain a close relation with the original problem. In this talk, we will give a general survey about the link between relaxation procedure and applications, with a particular attention to examples. Furthermore we will discuss the case when some pathologies in the relaxation procedure arise and the relation between this phenomenon and the Clarke's Hamiltonian Inclusion

Sensitivity-based multistep feedback MPC

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Abstract

In model predictive control (MPC) applications, an optimization problem is solved at every sampling instant to determine a sequence of input moves that controls the current and future behavior of a physical system in an optimal manner. There is a need to accelerate the solution of the optimization problem when applying MPC to fast systems wherein sampling frequencies are high. Solving the optimization problem on an embedded hardware also presents a challenge since it has to be computationally efficient and reliable.

Sensitivity-based multistep feedback is a feedback strategy that allows performing optimization less often and incorporates sensitivity analysis to update the entries of the multistep feedback to maintain the robustness of the scheme by accounting for uncertainties affecting the system had it run in open-loop.

From an implementational point-of-view, we discuss how this feedback strategy can significantly reduce the time, cost and energy consumption needed to compute the control action while fulfilling performance expectations.

New approach for stochastic target problems with state constraints

Athena Picarelli
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Abstract

This work is concerned with stochastic optimal control for a running maximum cost. A direct approach based on dynamic programming techniques is proposed leading to the characterization of the value function as unique viscosity solution of a second order Hamilton-Jacobi-Bellman equation with oblique derivative boundary condition. This work is strongly motivated by the will of developing an alternative and numerically effective way for dealing with state-constraints in stochastic control and in particular in stochastic target problems. In fact an optimal control problem with a cost depending on the running maximum will arise in the characterization of the backward reachable set for a system of controlled stochastic differential equation applying the level set approach together with an exact penalization technique.

LBFGS method, test results

Sonja Rauski,
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Abstract

Discretized optimal control problems typically lead to high dimensional nonlinear optimization. To solve these problems we need to determinate the Hessian matrix, which requires a lot of memory consumption. We present an efficient algorithm for calculating the Hessian matrix, namely, the limited memory BFGS method (LBFGS). Afterwards the results from the test on a software library for mathematical nonlinear optimization WORHP are shown. In the test LBFGS is compared with other methods for Hessian matrix approximation. Furthermore example from robotic application is presented.

Numerical Schemes for Hamilton-Jacobi Bellman equations

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Abstract

Our aim is to develop accurate schemes for Hamilton Jacobi Bellman Equation with discontinuous solutions.

Recently an anti-dissipative scheme has been proposed in order to deal with this kind of solutions. They typically require a projection step onto a discontinuous reconstruction space, but this choice seems to be rather unusual when the solution is regular. On the other hand Semi-Lagrangian schemes are well performing on Lipschitz continuous solution. Then a natural idea is to couple the two schemes to have at the same time an accurate resolution on the jumps and on the smooth region.

Theoretical and numerical concepts used for conservation laws can be adapted to Hamilton Jacobi equations. Using this observation we will propose an extension of Essentially non-oscilatory scheme based on subcell resolution. Finally, we will address a possible development of Discontinuous Galerkin method for the Hamilton-Jacobi Bellman equations.

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Local regularity of the value function of Mayer's problem with differential inclusions

Teresa Scarinci
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Abstract

The goal of this talk is the study of the local regularity of the value function of a Mayer problem, where the state equation is given by a differential inclusion of the type:

$$\begin{aligned} &\dot{x}(t) \in F(x(t)), \text{ for a.e. } t \in [t_0, T], \\ &x(t_0) = x_0, \end{aligned} \tag{3}$$

where F is a multifunction subject to suitable structural assumptions. The value function of such a problem is locally semiconcave in $(-\infty, T] \times \mathbb{R}^n$ (see [2]) and satisfies, in the viscosity sense, the Hamilton Jacobi equation:

$$-u_t + H(x, -u_x) = 0, \tag{4}$$

where H is the Hamiltonian defined by:

$$H(x,p) = \sup_{v \in F(x)} \langle v, p \rangle. \tag{5}$$

On the other hand, it is well known that V fails to be everywhere differentiable, in general. Even when V is differentiable at a point (t,x), this dress not yields that V is smooth in a neighborhood of (t,x). When equation (4) is associated to a Bolza problem in the calculus of variations, a sufficient condition for the local regularity of V has been recently proposed in [1] . Such a condition requires, among others things, the existence of a proximal subgradient of V at (t_0, x_0).
In this talk we will recover a similar result for a Mayer problem associated to (3). An essential step of the analogies is the proof of a so-called *proximal subgradient inclusion*, which is also a result of indipendent interest. The main technical difficulty of the recent problem, compared with the situation studied in [1] ,is the fact that the Hessian matrix is only positively semidefinite in the case of Mayer's problem. Indeed, for a geometric Hamiltonian we have that:

$$H_{pp}(x,p)p = 0, \text{ for all } x \in \mathbb{R}^n, p \in \mathbb{R}^n \setminus \{0\}. \tag{6}$$

Nevertheless, we will prove that if $H_{pp}(x,p)$ is positive definite in all direction that are orthogornal to a vector p, then we can derive the local smoothness of V.

References

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Necessary optimality conditions in control theory and regularity of minimizers

Daniela Tonon
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Abstract

We discuss the Bolza and the Mayer problems arising in optimal control which are of great interest in all models involving dynamic optimization. The main frame is to minimize a functional under a dynamic constraint which involves controls, that is parameters depending on time. It is well known that every strong local minimizer of these problems satisfies the Pontryagin maximum principle. In the absence of constraints qualifications the maximum principle may be abnormal, that is, not involving the cost functions and therefore it provides useless information. We provide sufficient conditions for normality and apply them to guarantee the non occurrence of the so-called Lavrentieff phenomenon. When an optimal control is singular, second-order optimality conditions are necessary in order to identify optimal trajectories. To this aim we provide a generalization of the Goh and the Legendre-Clebsch conditions.

Collision Avoidance for Driver Assistance Systems

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Abstract

Collision avoidance for driver assistance systems is an important and active field of research in car industry. Over the years many passive, semi-active or active safety systems and driver assistance systems have been developed with the aim to reduce the number of casualties in traffic accidents or to prevent accidents. In addition to the required technical devices and intelligent software systems, algorithms play a crucial role in solving this problem: optimal control problems are often used for investigating future collisions and to provide possible escape trajectories. A model which is commonly chosen in the automobile industry for basic investigations of the dynamical behavior of cars is the 7 states single track model. However in the simulations, a 4 states point mass model or a 5 states kinematic model, which are easier to handle numerically, are often taken into account. We tested two different softwares for such models and collect some numerical calculations. The package OCPID-DAE1 with a Fortran 90 interface is designed for the numerical solution of optimal control problems and parameter identification problems. The ROC-HJ Solver for solving Hamilton-Jacobi Bellman equations can be used for reachable sets computations and optimal trajectory reconstruction. A parametric sensitivity analysis is adopted in order to investigate the influence of inaccurate sensor measurements. Hence we define robust reachable set and perturbed reachable set and we discuss a characterization for the possible error around a given initial data.

Stability of Economic NMPC for Periodic Systems

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Abstract

Model Predictive Control (MPC) schemes are commonly using reference-tracking cost functions, which have attractive properties in terms of stability and numerical implementation. However, many control applications have clear economic objectives that can be used directly as the NMPC cost function. Such NMPC schemes are labelled Economic NMPC. Unfortunately, Economic NMPC schemes suffer from some drawbacks. In particular, stability results for economic NMPC are still very sparse. A Lyapunov function for Economic NMPC was first proposed in Diehl2011 for problems having a steady-state optimum. In Zanon2013 (submitted to CDC) this approach is generalized for periodic systems.